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MTR-2330

A FLEXIBLE HARDWIRED FAST FOURIER TRANSFORM DIGITAL PROCESSOR

G. C. O'Leary
E. A. Palo

MARCH 1973

Prepared for

DEPUTY FOR PLANNING AND TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts



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FOREWORD

This report has been prepared by The MITRE Corporation under Project 702B of Contract F19(628)-71-C-0002. The contract is sponsored by the Electronic Systems Division, Air Force Systems Command, L. G. Hanscom Field, Bedford, Massachusetts.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.



LAWRENCE F. SOKOLOWSKI, Captain, USAF
Directorate of Development Engineering
Deputy for Planning and Technology

ABSTRACT

A hardwired digital processor based on the Cooley-Tukey algorithm is presented. A laboratory prototype has been built with two modes of operation. As a cascade fast Fourier transformer, it can simultaneously calculate transforms of two independent, continuous data streams at word rates in excess of 3 MHz. As a nonrecursive digital filter, it can produce filter impulse responses of up to 32 points. The digital filter also operates from data source with word rates exceeding 3 MHz.

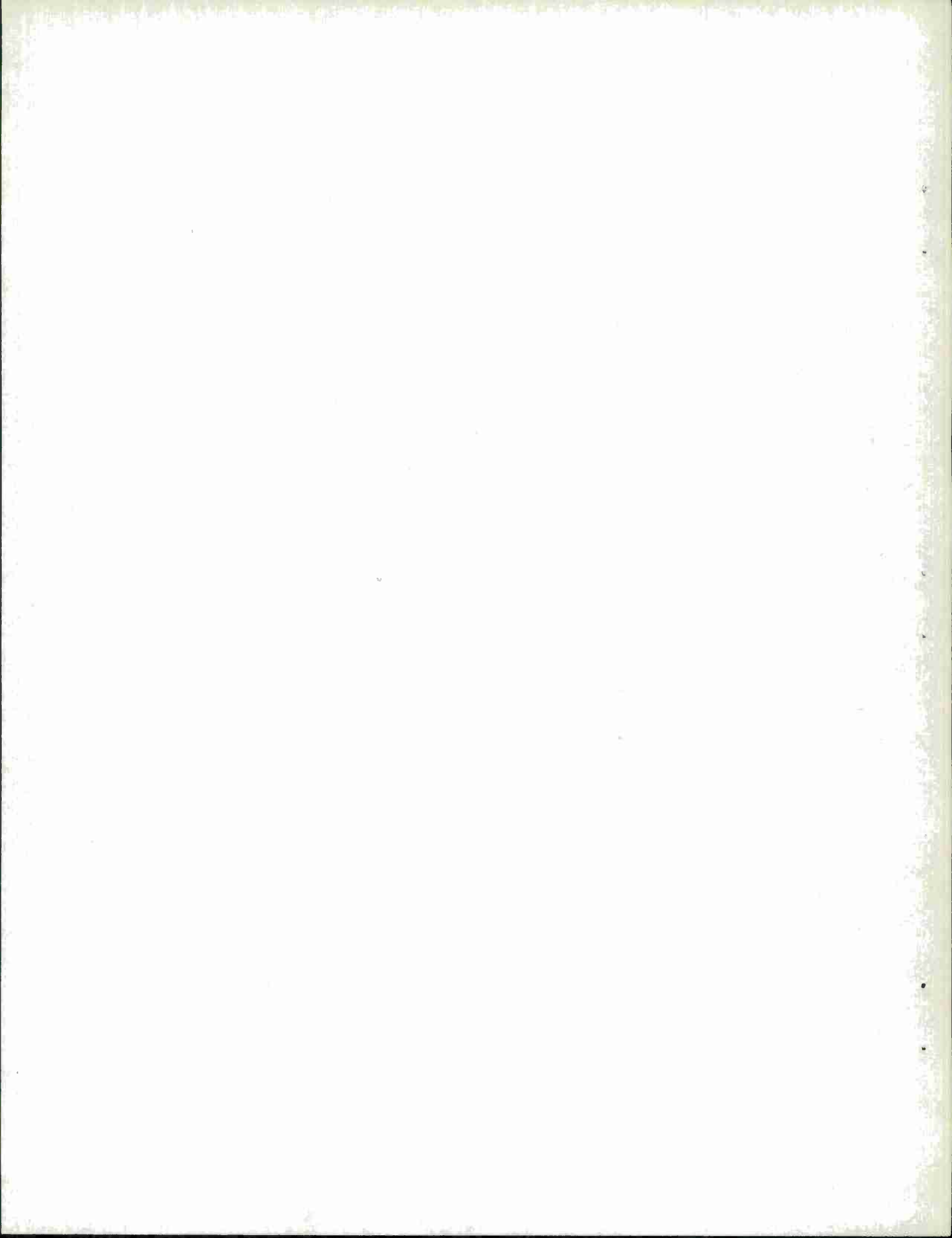
The processor has been integrated into a system with other signal processing components including a small general purpose computer. Laboratory demonstration of the processing system as a spectrum analyzer and as an adaptive filter for distortion correction is discussed.

ACKNOWLEDGMENT

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SECTION I

INTRODUCTION

This paper describes a digital processor based on the fast Fourier transform (FFT) algorithm. The paper begins by describing the design theory for a fast Fourier transformer and continues with the development of two versions for a cascade FFT each capable of transforming simultaneously two independent data blocks. The cascade organization permits designing FFT processors for high data rates.

The hardware design of a laboratory prototype is then discussed. The resulting processor has two modes of operation: As a 512 point FFT, it can simultaneously transform two independent data sets. As a non-synchronized digital filter, it can be used in applications requiring up to 32 points of impulse response. Both modes operate at word rates in excess of 3 MHz, making possible their use as real time, wideband processors.

Also covered is the integration of the laboratory prototype with other signal processing components into a flexible laboratory system with real time spectrum analysis and adaptive filtering capabilities on bandwidths approaching 3 MHz.

SECTION II

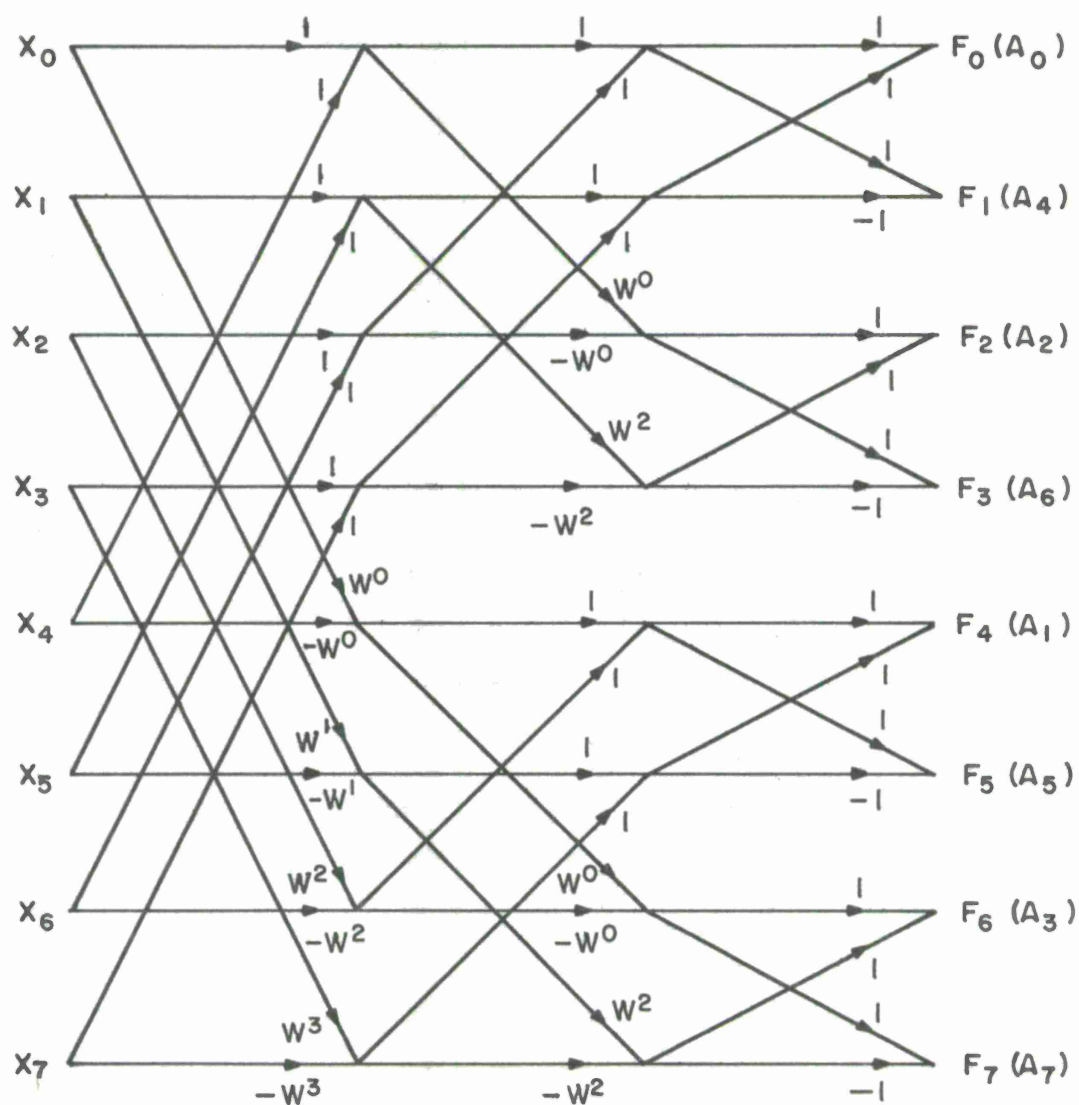
THE ALGORITHM

The Fast Fourier Transform (FFT) algorithm provides an efficient procedure for calculating the discrete Fourier transform (DFT) of a data set containing $N (=2^k)$ points ⁽¹⁾. The algorithm requires a total number of operations proportional to $N \log_2 N$, whereas the straightforward approach requires a number of operations proportional to N^2 ; for large N the savings are significant. The algorithm can be generalized to a radix other than 2, but this discussion will be confined only to transforms using radix 2.

The object is to calculate the discrete Fourier transform of the series x_0, x_1, \dots, x_{N-1} .

$$A_n = \sum_{k=0}^{N-1} x_k e^{-j \frac{2\pi}{N} kn} \quad n=0,1,\dots,N-1 \quad (1)$$

The decimation in frequency form of the algorithm is considered here. A similar development using decimation in time is possible. The flow chart for an 8 point transform using decimation in frequency (Figure 1) can be implemented by the design shown in Figure 2⁽²⁾. A convenient property of the decimation in frequency algorithm is that the last k stages constitute a 2^k point FFT. Extending this design for larger transform capability means adding similar stages at the input.



$$W = e^{-j\frac{2\pi}{N}}, N = 8$$

$$A_n = \sum_{k=0}^{N-1} X_k e^{-j\frac{2\pi}{N}kn} \quad n = 0, 1, \dots, N-1$$

Figure 1. Flow Graph of 8 Point FFT for Decimation in Frequency

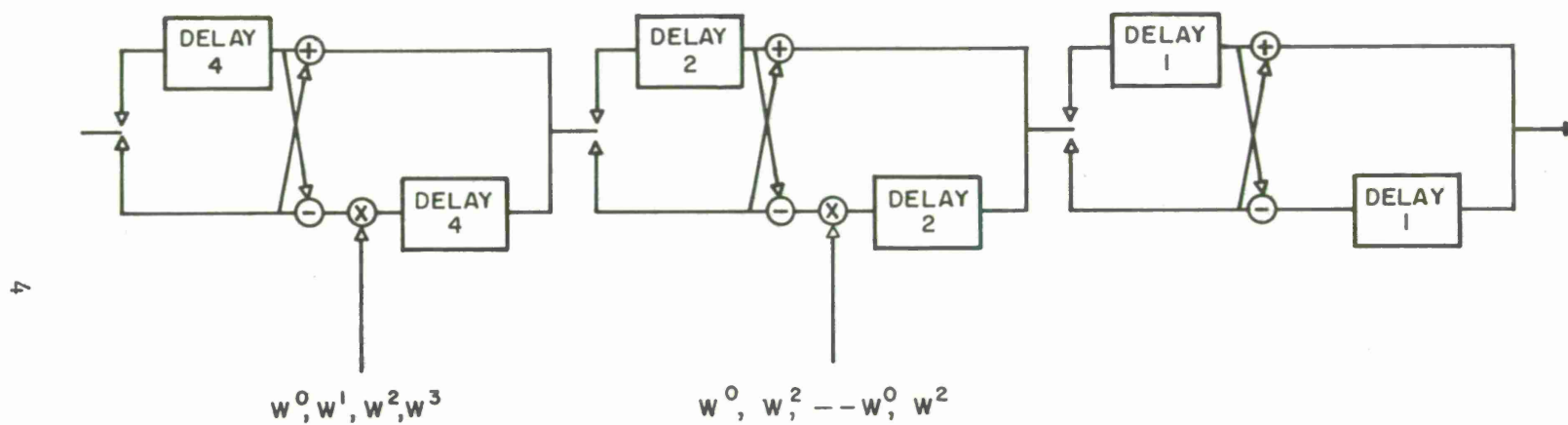


Figure 2. FFT Processor for $N=8$. Inputs are in Natural Order. Outputs are in Bit Reversed Order.

Study of the data flow for the design of Figure 2 reveals that all circuitry is idle half the time. This fact is exploited to advantage by rearranging each stage to have the canonic form shown in Figure 3. The increase in complexity of the rearranged interstage switch is offset by the savings in delay line storage and by the capability of computing two transforms simultaneously. Cascading k stages of the form shown in Figure 3 results in a 2^k point dual input FFT. If two continuous data streams are presented, one to the top input and the other to the bottom input of the first interstage switch, the transforms of both data streams will be calculated and separated at the output. The frequency points appear in bit reversed order at the output, half from each input on the top path, the other half on the bottom. Also, if the spectrum of the input is considered to be centered at zero frequency, all points at the top transform output represent plus frequencies while the transforms points out the bottom are the negative frequency components.

BIT REVERSED INPUTS

Because of the interest in the development of digital filters using FFT techniques, it was necessary to arrive at a companion FFT design that accepted input data in bit reversed order. A rearrangement of the flow graph for natural order inputs (Figure 2) allows a bit reversed input order and a natural output order.⁽¹⁾

A rearranged implementation similar to the previous one is possible, and its canonic form is shown in Figure 4. Here data to

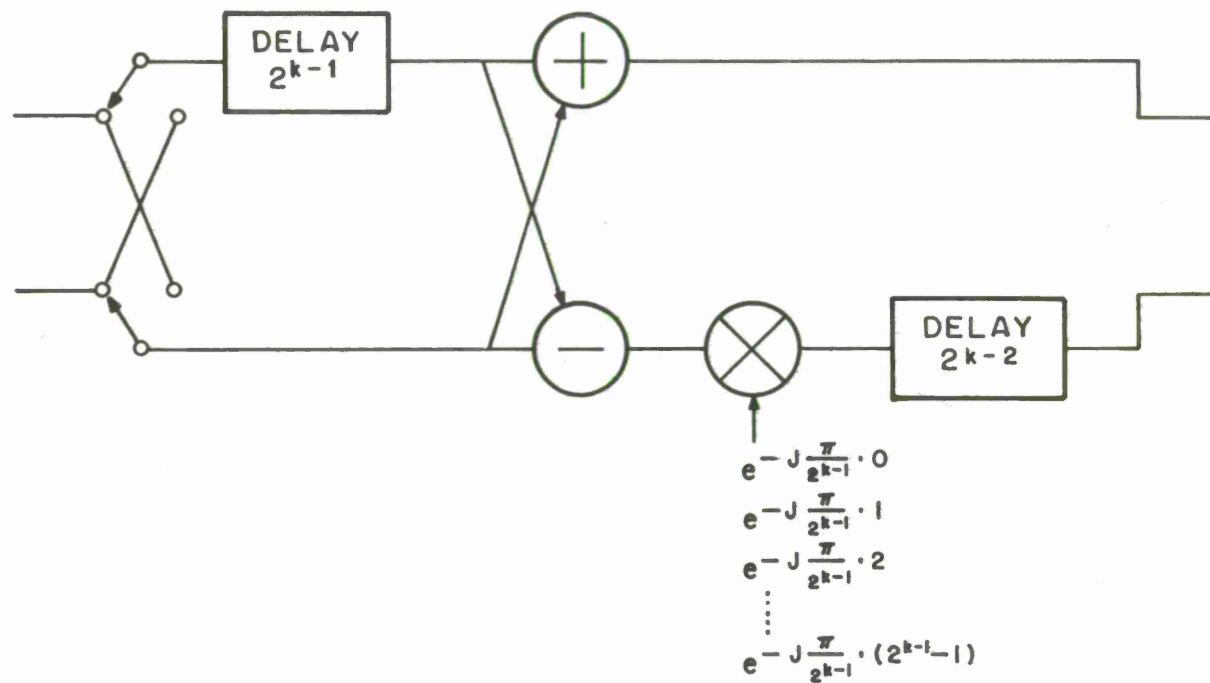


Figure 3. k th Stage from Output - Canonic Form for Natural Input Order.

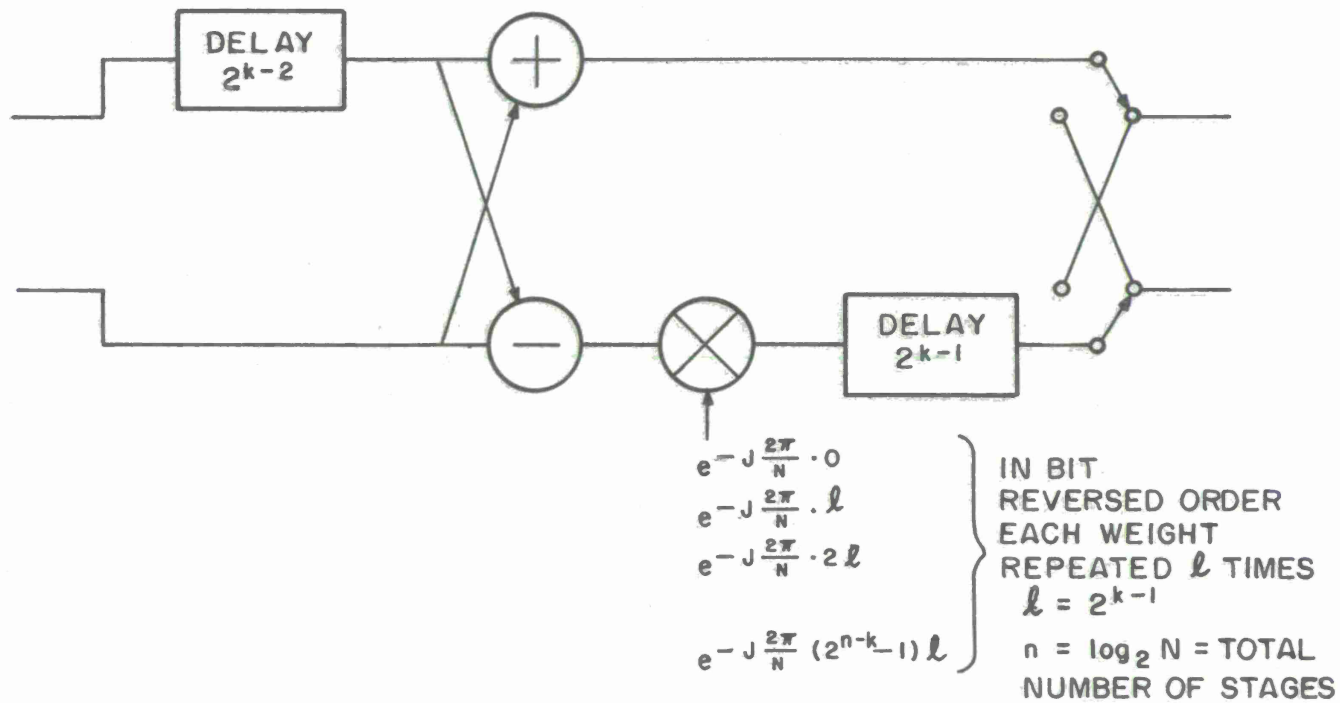


Figure 4. k th Stage from Input - Canonic Form for Bit Reversed Input Order.

be transformed is input on the top and bottom paths and ends up in serial format after the final stage switch output. The more complicated interstage switch and the corresponding decrease in delay line storage have been utilized here, too. And again, circuitry idle half the time is captured by a second, independent data input block offset in time by 2^{k-1} points. A 2^k point, dual input transform for bit reversed inputs is developed again by cascading k stages.

The implementation for transforming two data sets each of $N(=2^k)$ points with either FFT design is developed by cascading k similar stages from the proper canonic form. In either case the resulting complexity is k interstage switches, k arithmetic sections (addition-subtraction), k multipliers, N weighting coefficients, and $\frac{3}{2}N$ words of delay line storage. Either design will calculate the inverse DFT by using the conjugate Fourier weights.

DIGITAL FILTER

It has been shown that performing discrete convolutions by transform methods can yield computational savings ⁽³⁾. The inverse DFT of a function formed by the product of the DFT's of two data sets is the circular convolution of the two data sets. Continuous digital filtering can be realized with this technique if care is taken to make the circular convolution appear as a normal aperiodic convolution. This is accomplished by using N point time signals having $\frac{N}{2}$ data points followed by $\frac{N}{2}$ zeros. For continuous signals this is done by breaking the input into two separate sets each

containing blocks of $\frac{N}{2}$ data points padded out with $\frac{N}{2}$ zero points and staggered by $\frac{N}{2}$ data points. This is illustrated in Figure 5.

Each signal x_T and x_B can be separately passed through a filter h_k described by $\frac{N}{2}$ points of impulse response (padded with $\frac{N}{2}$ zero points) and the separate responses are added to yield the result of x_k passed through the same filter. Using transform techniques this requires two separate transformers, two multipliers to form the product of the frequency characteristics, and two separate inverse transformers. Shown in Figure 6 is such a filter. Note that all transform operations are N points even though only $\frac{N}{2}$ data points are utilized (the $\frac{N}{2}$ zero points must be considered). This filter will give the correct output for the convolution of the input x_k , unlimited in extent, with the filter response h_k , which must be limited to $\frac{N}{2}$ points to avoid the aliasing of the circular convolution.

The dual input cascade transformers discussed in the previous section are ideal for the digital filter implementation shown in Figure 6. Advantage can now be taken of the two transform capability. If the data x_k is input to the top path of the transformer, and the bottom path is constrained to zero, and the switch is directed to the feed through position, then the data blocks, x_T and x_B , of Figure 6 are formed as the two independent inputs to the transformer. Elimination of the switch and delay line at the input stage will also accomplish the x_T and x_B data blocking.

When employed in this fashion, the outputs of the cascade trans-

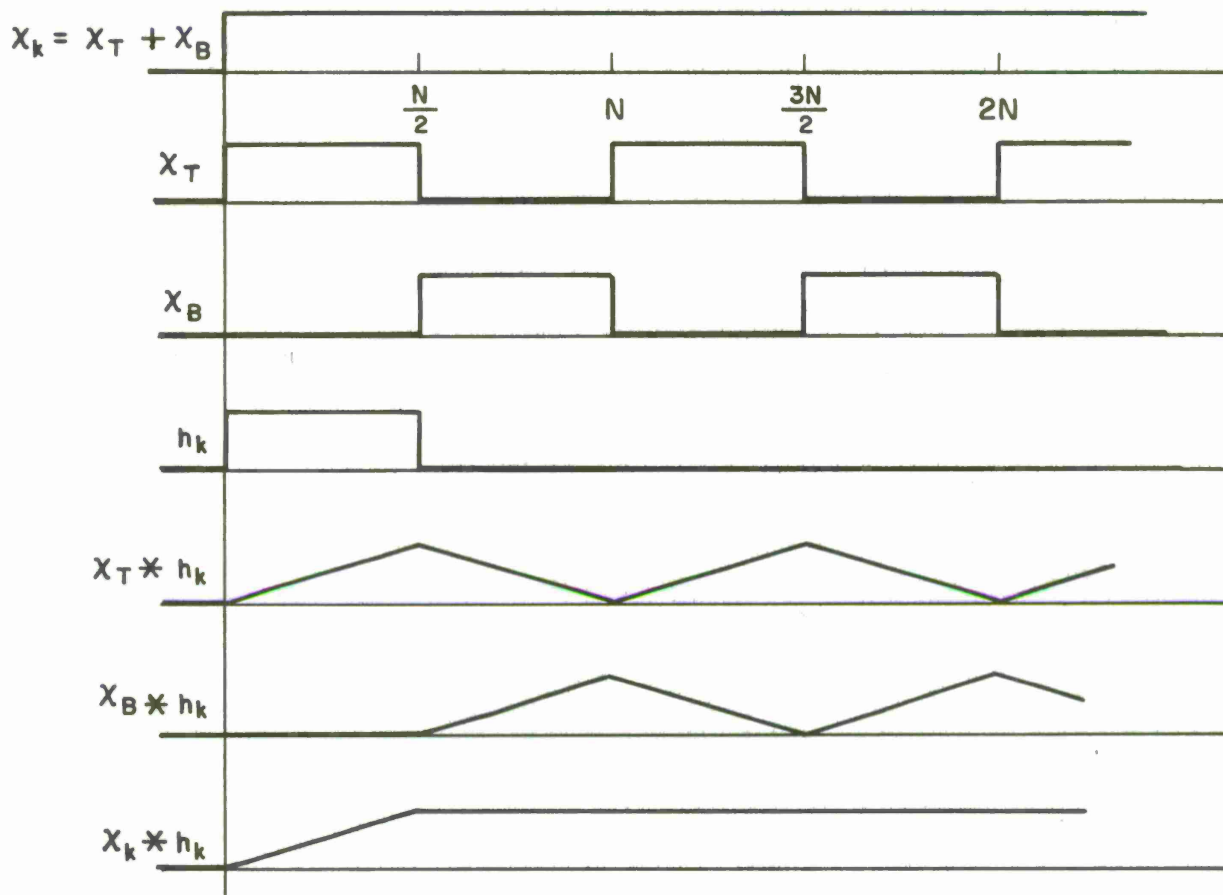


Figure 5. Data Blocking for Continuous Filtering

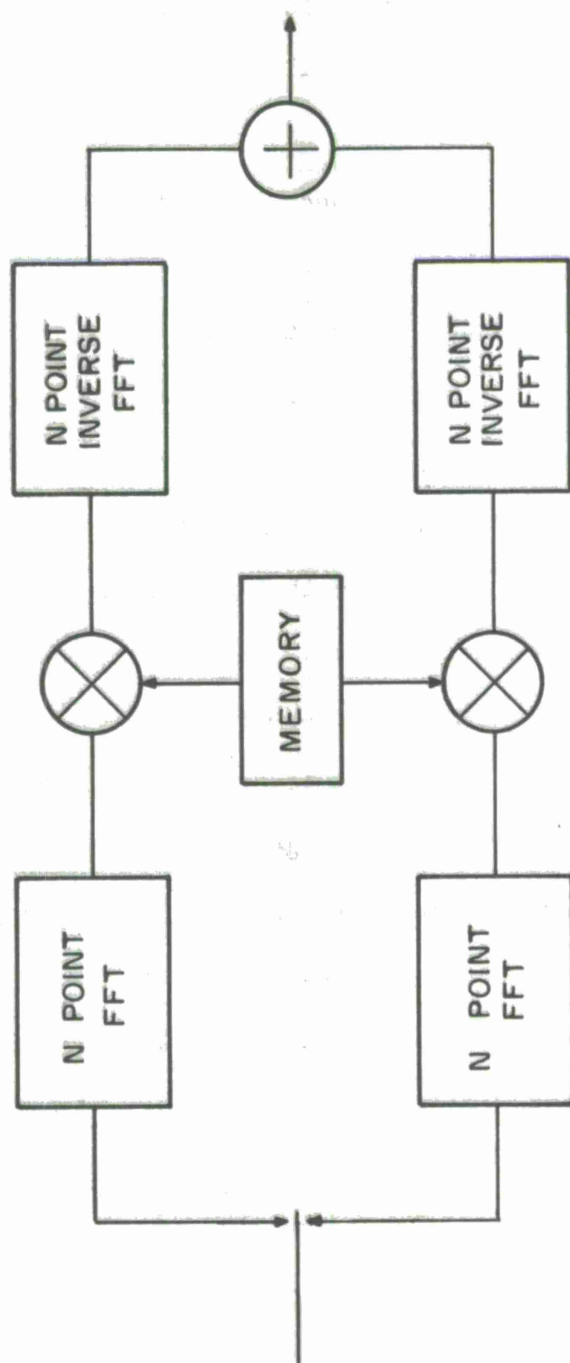


Figure 6. Continuous Filter Using FFT

former have the data points of the transform of x_T followed by those of x_B . Two multipliers at this point form the product of these transforms with the transform of the filter response h_k stored in the memory.

The outputs of the multipliers become the inputs for the cascade inverse transformer of the type already described which accepts bit reversed data inputs. Now the inputs are bit reversed frequency points and outputs natural ordered time points. The switch at the final stage output is replaced by an adder which sums the responses due to the x_T and x_B inputs. Thus, a digital filter is formed which operates continuously on the input signal x_k with no synchronism required between x_k and h_k . The construction of a filter h_k described by $\frac{N}{2}$ points of impulse response requires an N point dual input FFT (with modified input switch and delay structure), 2 complex multipliers, N words of storage for the transform of h_k , and an N point inverse dual input FFT capable of accepting bit reversed inputs while outputting natural order time points. The filter characteristic h_k can be altered by simply entering new transform values into the memory.

SECTION III

THE IMPLEMENTATION

The following constraints were placed on the laboratory prototype processor design: 1) In addition to its spectrum analysis capabilities, it should have significant impact in the area of adaptive digital filtering. 2) It should be developed from available components. 3) It should be capable of Megahertz processing rates. 4) It should not be unwieldy in size. 5) It should show a high degree of modularity with consideration for the future advances in large scale integration.

Ten bit, two's complement arithmetic is used for all operations. Complex words are described by 20 bit rectangular coordinates. Overflow in the arithmetic is detected but automatic correction is generally not possible in the cascade processor. Interstage scaling, as a front panel option, is incorporated to permit optimization of the dynamic range and to control overflows.

A typical stage for either type of transformer discussed requires five basic units: An interstage switch, delay line storage, and arithmetic section, a multiplier, and Fourier weighting coefficient generation or storage. The complexity of each of these items depends basically on two system parameters: Operating speed and digital word length. A preliminary design of critical components indicated that, with a 10 bit word length, 2 to 4 MHz maximum operating rates

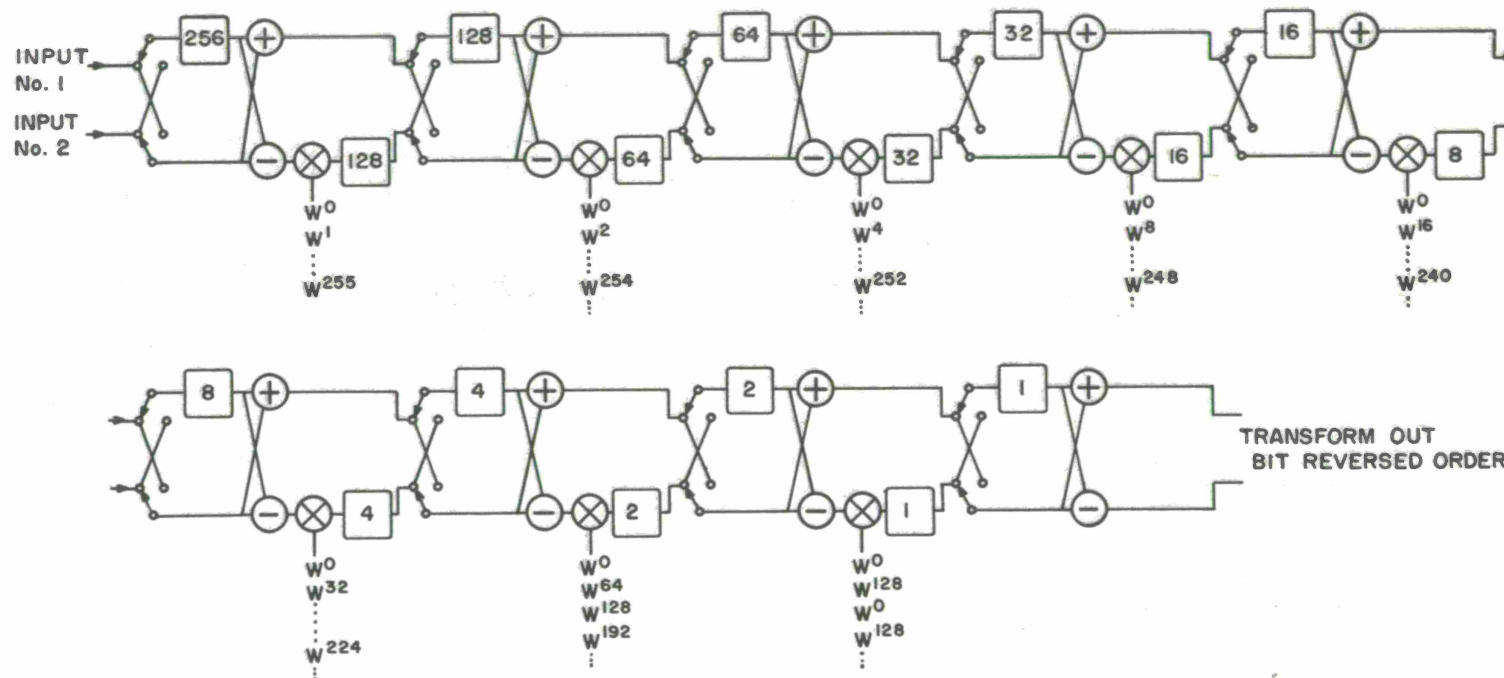
could be expected from the multipliers, the longer delay line storage, and weight storage.

The resultant size of nine stages (512 transform points) for the dual input, cascade FFT processor provides reasonable size and complexity. Transform lengths of any power of two through 512 points are possible with this equipment. Also, should larger transforms be desired, additional stages could be added along with additional timing and control. A block diagram of the nine stage FFT is shown in Figure 7.

The digital filter design consists of a six stage dual input FFT, two multipliers and storage, and a six stage dual input inverse FFT. The six stage FFT is exactly the last six stages of the 512 point dual input FFT. The six stage inverse FFT for bit reversed inputs is a separate subsystem, but the design permits borrowing some computation modules from the three unused stages of the 512 point FFT. The block diagram of this twelve stage digital filter is shown in Figure 8.

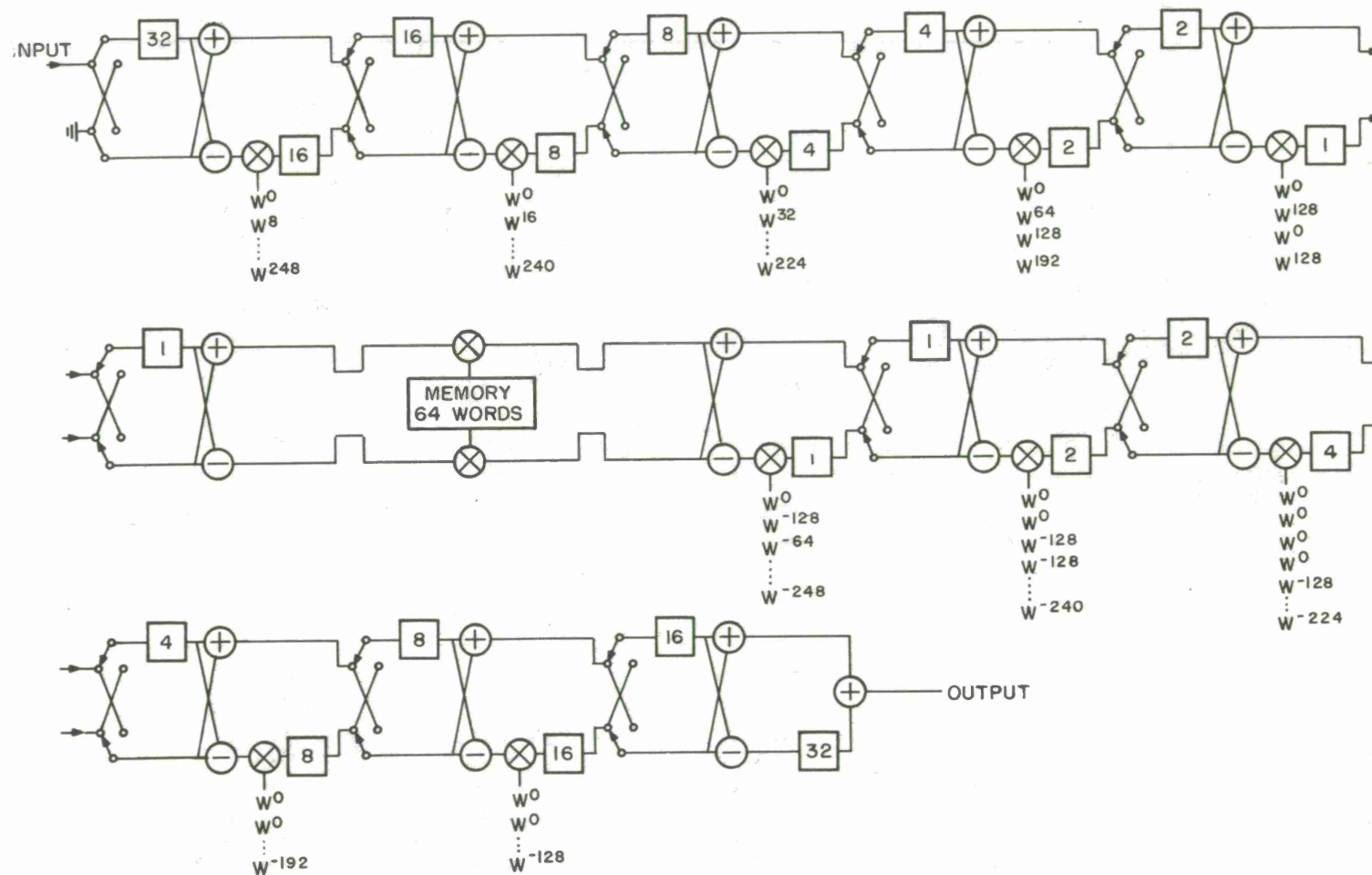
INTERSTAGE SWITCH

The interstage switch routes the top and bottom outputs of one stage to the top and bottom inputs of the next upon control from the central timing. A shift right option is added to the switch to divide all inputs by two with no rounding. There is a negative roundoff error introduced by this scaling, but it was accepted to avoid the use of the additional components necessary to round the



$$W = e^{-j\frac{2\pi}{N}}, N = 2^9 = 512$$

Figure 7. Nine Stage - 512 Point - FFT



$$W = e^{-j\frac{2\pi}{N}}, N = 512$$

Figure 8. 12 Stage Digital Filter

answer. This scaling option does not compensate for all overflow possibilities in a stage (4). The implementation for a 10 bit word is carried out with standard TTL multiplexing devices on a 2 1/2 inch X 7 inch circuit card. Two cards are needed per stage.

DELAY LINE STORAGE

The nine stage laboratory prototype employs delay line lengths ranging from 1 word to 256 words. Delay lines are designed for 10 bit wordlengths using available TTL and MOS devices.

The 1, 2, and 4 word delay lines are constructed from a common TTL D-type flip-flop. The 8, 16, and 32 word length delay lines use TTL 8 bit long shift registers. The shift rates and propagation delays are substantially better than the system requires.

The 64, 128, and 256 word delay lines are developed from the same dual 64 bit dynamic, P channel, MOS shift register. The two phase clocking is developed on the circuit board from available hybrid clock drivers. A TTL D-type input register was added to maintain the edge triggered input clocking compatible with the TTL delay lines. The resultant shift rates, limited by the drive capabilities of the clocking, approach 4 MHz. A 64 word delay line is contained on a 2 1/2 X 7 inch circuit card, while the 128 and 256 word delay each occupies a 5 X 7 inch power card. Power dissipation at a 3 MHz shift rate measures less than 5 milliwatts per bit including clocking.

ARITHMETIC SECTION

The basic function of this circuitry is to perform the addition and subtraction of complex words as shown in the block diagrams, and to provide the additions necessary to complete the complex multiplications. The implementation is carried out with a combination of TTL 2 and 4-bit adder packages, using 10 bit two's complement operations. The output of the 10 bit addition is clocked into a one word delay register to compensate for the input register associated with the multiplier following the subtraction. Arithmetic overflow is detected, but with the cascade processor it can only signal that computations are in error. In general, to correct for the overflow the entire data block must be processed again.

Overflow detection operates on the basis that only when the input numbers have the same polarity can an overflow situation exist. The overflow is then indicated by the opposite polarity in the output word. Gating logic implements the overflow detection and strobes it into a latch where it is stored until cleared by a source that is external to the arithmetic section.

A 10 bit adder with overflow detection was made a part of the arithmetic section, since the formation of the complex product by the multiplier following the subtraction requires two additions. The resultant circuit is contained on a 2 1/2 X 7 inch circuit card using solderless wrap interconnections, and two cards are used in each stage.

COMPLEX MULTIPLIERS

Each multiplication for complex arithmetic in rectangular coordinates requires a total of four basic multiplications and two additions. Eliminating the trivial case of multiplication by unity, the nine stage FFT requires eight complex multipliers, while the digital filter needs twelve.

Some hardware savings were made by developing special purpose, hardwired multipliers for those stages requiring only a few weights. As a result, the multiplier design was divided into three separate areas: one general purpose and two special purpose multipliers.

One of the hardwired complex multipliers is configured to multiply its input by 1 , $e^{-j\frac{\pi}{2}}$ or $e^{j\frac{\pi}{2}}$ upon command from the timing. Both $\pm e^{j\frac{\pi}{2}}$ phase weights are necessary to supply the weights for calculation of either the discrete transform or the inverse discrete transform. Implementation is carried out by gating and multiplexing logic. The other special purpose multiplier is somewhat more involved. The necessary coefficients multiplying the input are 1 , $e^{-j\frac{\pi}{4}}$, $e^{-j\frac{\pi}{2}}$, and $e^{-j\frac{3\pi}{4}}$ for the discrete transform, and 1 , $e^{j\frac{\pi}{4}}$, $e^{j\frac{\pi}{2}}$, $e^{j\frac{3\pi}{4}}$, for the inverse transform. The same technique for multiplexing logic is used with addition of a hardwired $\frac{\sqrt{2}}{2}$ multiplier, which is done by the parallel additions of the input word shifted as indicated by ones in the binary representation of $\frac{\sqrt{2}}{2}$.

The basic multiplier used for the rest of the multiplications

accepts a pair of 10 bit two's complement numbers and outputs their product as a two's complemented number.

The multiplier forms the partial products in parallel. The partial products are then summed in a tree of parallel adders. The necessary correction factors required in two's complement arithmetic when one or both of the numbers are negative are also wired into the hardware.

Since 10 bit words are desired throughout this system, only the most significant ten bits are accepted as outputs. Care is taken to round this answer.

The implementation of the basic multiplier made maximum use of TTL 2 and 4 bit adder packages to perform the additions required by the word organized adder tree. The sixty integrated circuits are packaged, as shown in Figure 9, on a single 5 X 7 inch circuit card. Fabrication of thirty-two such multipliers was accomplished by automated solderless wrap techniques.

Dynamic testing of all multipliers was done with the aid of a small general purpose computer which tested all of the multiplier-multiplicand possibilities (2^{20}). A precise timing strobe was used to latch each answer and send it back to the computer for comparison with the correct product. All multipliers were checked for worst case propagation delays given all input possibilities. Worst case multiplication delays varied from 200 to 260 nanoseconds. Four multipliers plus the two additions on the arithmetic board were

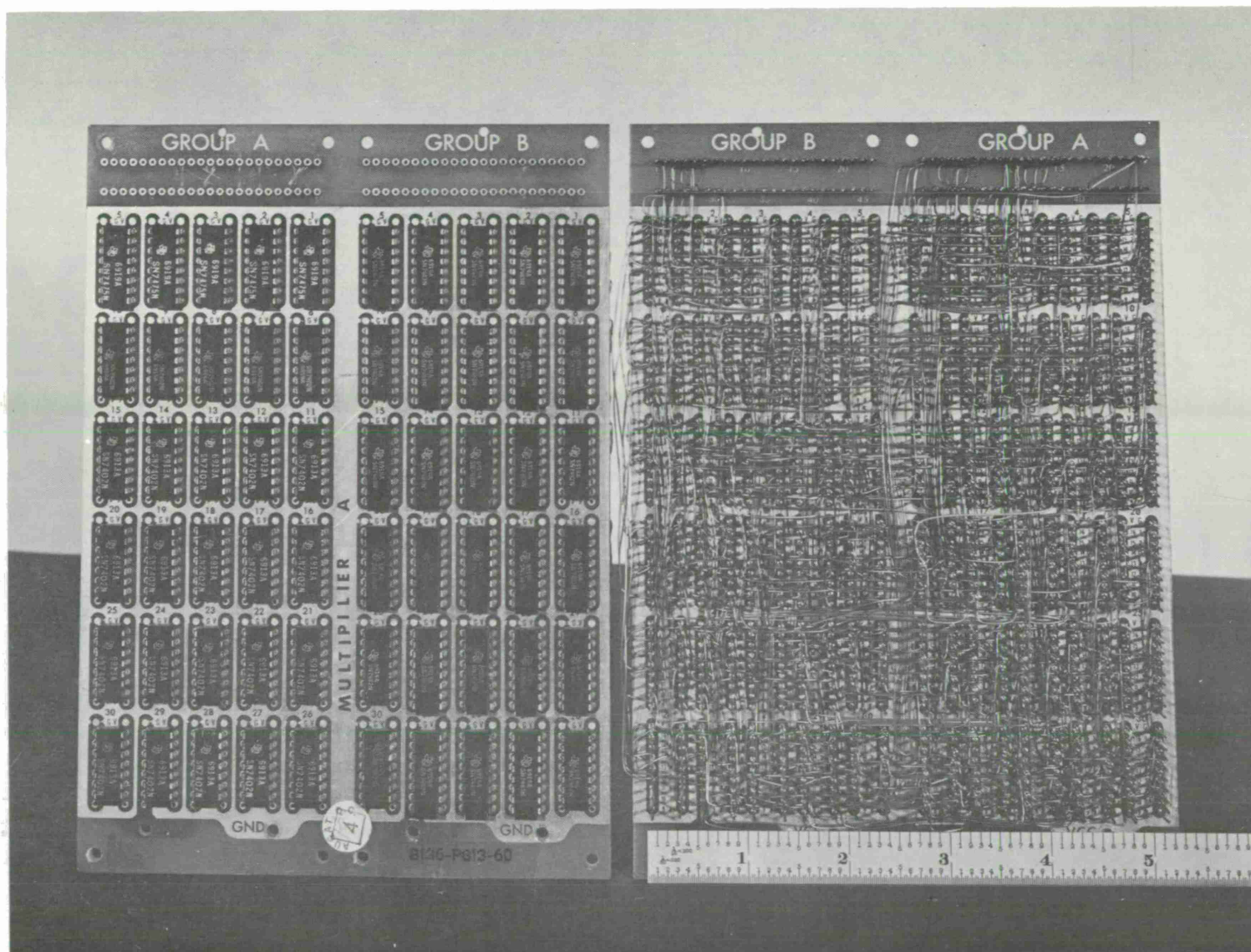


Figure 9. Front and Back Views of 10 Bit Two's Complement Multiplier

interconnected with the result that a full complex multiplication is completed in less than 300 nsec.

WEIGHT GENERATORS AND MEMORIES

The generation of the Fourier weight coefficients takes on several forms. No weights are needed at the stages employing the special purpose multipliers, since the weights are built into the hardware. The weights for the remaining stages are generated in two ways: Read-only-memory techniques, and straightforward synthesis using combinatorial logic. Also incorporated at each stage is the capability of conjugating the weights to provide the necessary coefficients for the complementary transform.

The stages requiring 8 and 16 distinct coefficients are each implemented using combinatorial logic on 2 1/2 X 7 inch printed circuit cards. The boards, which added the complex conjugate capability, are of similar size. Control signals are provided as 3 and 4 bit address lines from the central timing.

The weights for the input four stages of the FFT are stored in a braided transformer type read-only-memory (ROM). With an access time of 170 nsec and a cycle time of 300 nsec, it sets one of the speed limitations in the overall design. To conserve memory size and also use an available item, only the first 128 (of 256) weights (phase angles 0 to $\frac{\pi}{4}$) for the input stage are stored. The remaining 128 are generated from them. The final ROM unit contains 10,240 bits of storage.

The input stage of the companion design requires generation of 32 weighted coefficients, which is accomplished with TTL fusible link type read-only memories on a single 2 1/2 X 7 inch circuit card.

In addition to those weights that are needed for the multipliers internal to the FFT, filter characteristic weights are needed for the two multipliers in the digital filter. Since it is desired to alter the filter characteristics in the adaptive filter applications, a random access memory is used. In addition to its control by the central timing, it can be accessed by an external source. The memory is organized for 64 twenty bit words. Writing into the memory can be done from an external source by random access addressing or under control of the internal system timing for applications involving correlations. The design utilizes 64 bit (16 words 4 bits per word), fully decoded, TTL random access memories, and the completed memory with its control logic was packed on a 7 1/2 X 7 inch circuit board.

THE TIMING

The overall system timing and clocking is controlled from a central timing unit, which basically consists of a ten stage counter and clock buffers. Since all operations in the nine stage FFT or twelve stage digital filter have periods equal to a power of two number of clock cycles, the flip-flops of the counter essentially provide all the control signals necessary. However, since each transformer stage has an extra delay register associated with each

multiplier and arithmetic section the controls required in each stage are out of phase with all others. This situation is compensated for by incorporating in the timing an adder circuit for each stage to add or subtract the necessary phase offset needed in the counter to properly synchronize the controls for the system. A reference for zero offset was chosen as the output of the FFT. The interstage switch control and weight generator addressing are taken directly for the adder outputs.

All master clocking is developed on the timing board from the external clock input and distributed to the four main sections of the system. There they are buffered further for increased fan out.

SECTION IV

SYSTEM INTEGRATION

FABRICATION

The circuit boards described in the previous section are mounted planar fashion on frames capable of accepting up to three 5 X 7 inch boards (Figure 10). The frames are mounted vertically in drawers which contain slots for fifteen frames. A solderless wrap backplane in each drawer provides for interconnection of the frames. Access to 10 bit words on the backplane for monitor and test capability is made available by consistent organization of the word bits, and the use of a special 10 pin adapter plug which fits over the wrap posts. The entire system is housed in four drawers that are slide mounted in a standard size rack.

The entire system is shown in Figure 11. The nine stage FFT is contained in the top two drawers plus part of the third one. The random access memory for the filter characteristic along with the two multipliers plus the input stage for the companion design complete the third drawer. The remaining five stages of the companion transformer occupy the fourth drawer. Six of the pull out frames that contain computational units are shared, depending on the systems configuration, between the input three stages of the nine stage FFT (drawer 1) and the input three stages of the companion transformer (drawers 3 and 4).

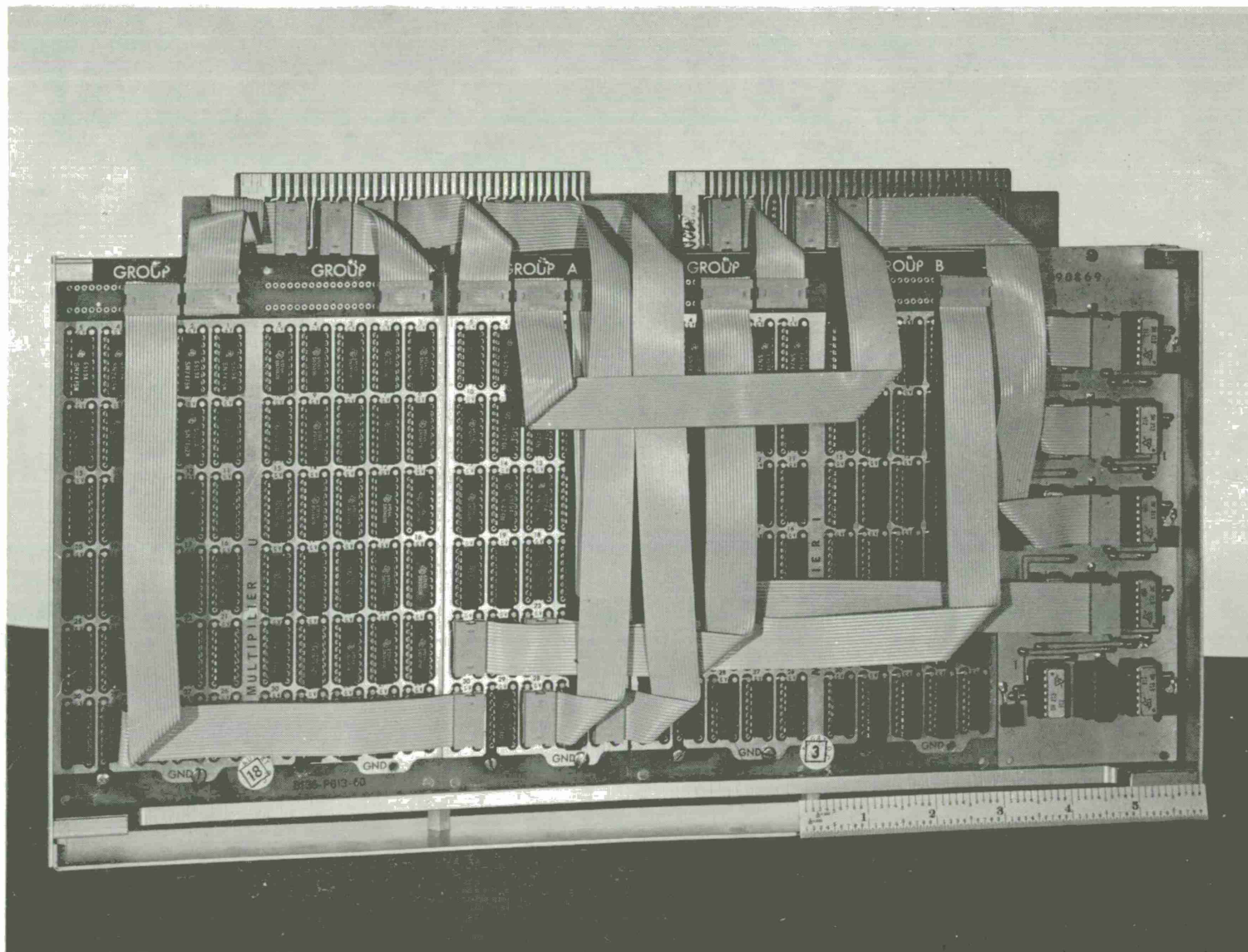


Figure 10. Computation Unit - 2 Multipliers, 1 Arithmetic Unit, 1 Interstage Switch

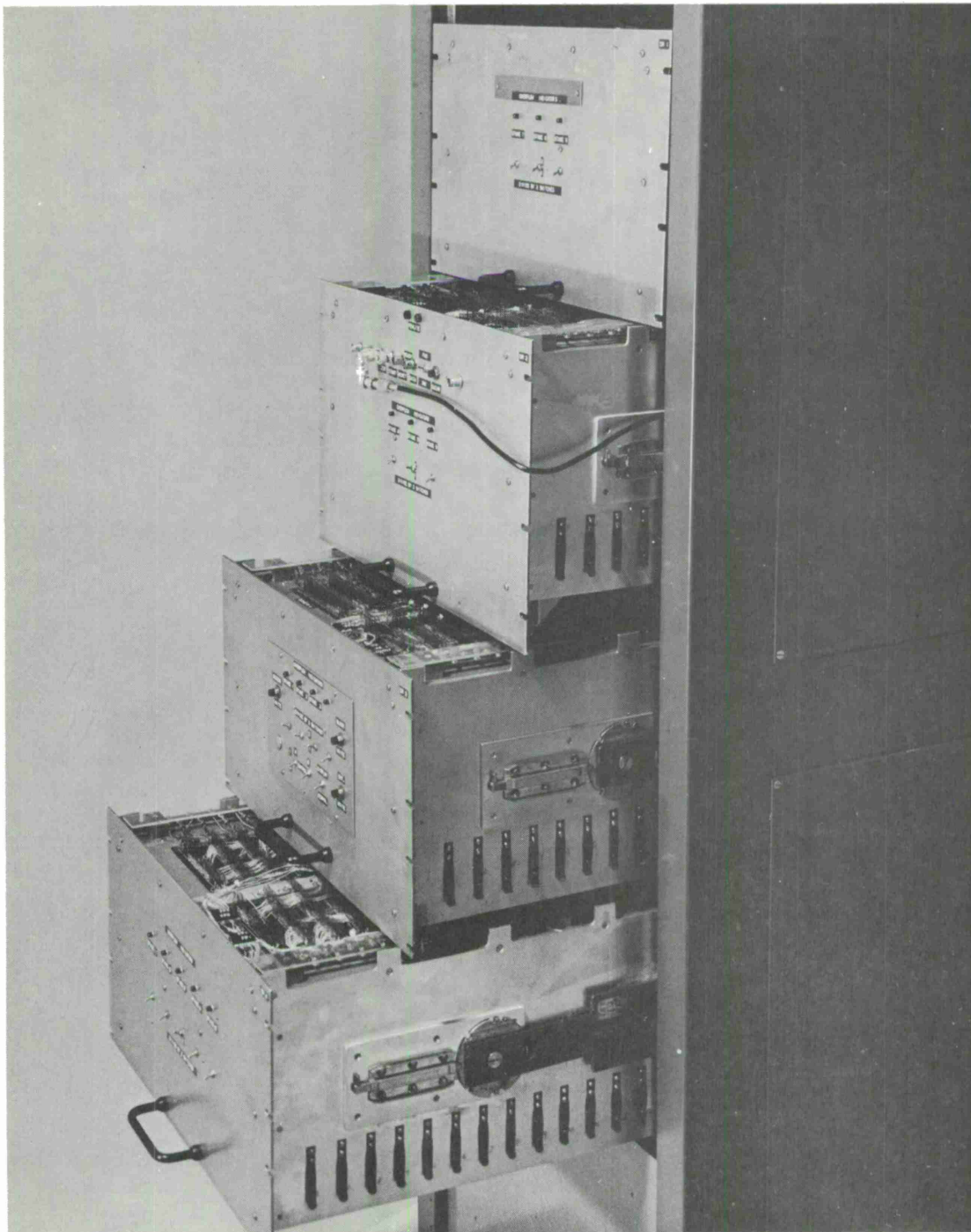


Figure 11. Complete Processor for Nine Stage FFT and Twelve Stage Digital Filter

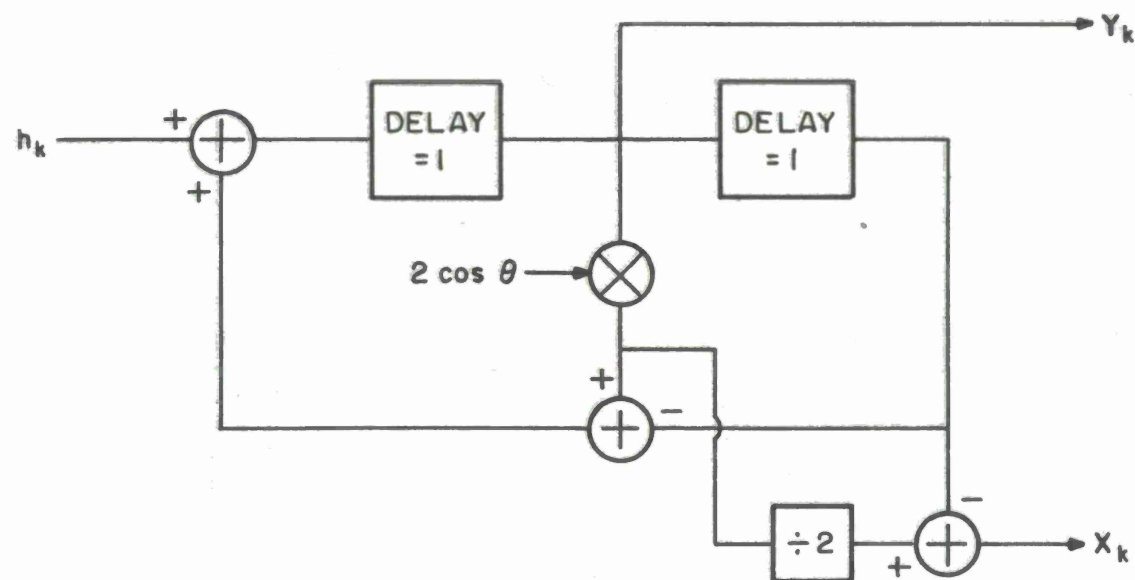
DESIGN VERIFICATION TESTS

The system testing was done using a simple recursive filter (Figure 12), which produces sinusoidal outputs when impulsed for a signal generator ⁽⁵⁾. The output "frequency" is controlled by the constant entered into the multiplier, which is a 10 bit two's complement device described earlier.

A computer simulation of the entire system was employed to provide a printout sheet of the actual numbers expected as a result of recursive filter inputs. Using the simulation results, the FFT and the digital filter configurations were debugged, and with a additional display aids dynamic operation at word rates in excess of 3 MHz was verified.

THE RESULTANT PROCESSOR

The integration of the FFT/digital filter with other components has resulted in a laboratory system with real time processing capabilities. The other critical elements available for the system are a general purpose computer and a digital multiplex switch. Peripherals for the 16 bit word computer included magnetic tape, disc, teletype, line printer, and output display screen. The digital switch allows the interconnection of various laboratory equipments to be controlled by its front panel switches or by software from the computer. System configuration changes are possible under software control at computer input-output (I/O) rates. Two of the computer I/O channels are connected to the switch--one for data



$$\text{IF } h_k = \begin{cases} 1 & k=0 \\ 0 & k \neq 0 \end{cases}, \text{ THEN } Y_k = \frac{\sin k\theta}{\sin \theta}$$

$$X_k = \begin{cases} 0 & k=0 \\ \cos k\theta & k \geq 1 \end{cases}$$

Figure 12. Recursive Filter

transfer and the other for system control. The devices connected to the switch and under its control included analog to digital (A/D) converters (up to 10 MHz conversion rates available with quadrature mixing and filtering), digital to analog (D/A) converters for display, a 1024 word (10 bits per word) buffer memory, plus the FFT and recursive filter.

Spectrum Analysis

The spectrum analysis capabilities of this equipment include both real time and non real time processing. When the nine stage FFT is joined by the high speed A/D converters, IF bandwidths approaching 3 MHz can be continuously analyzed with 512 resolution cells. The two transform capability is exploited by also inputting data from another pair of A/D converters, keeping all clocking common. The transformed data is output at a continuous 120 megabits per second, which poses a further processing problem depending on the application. Insertion of the buffer memory in front of the FFT will provide analysis of IF bandwidths approaching 10 MHz, but continuous analysis of the input is then sacrificed. More logically, the buffer memory is located after the FFT where it accepts selected transforms for subsequent entry into the computer. In this mode, continuous coverage of IF bandwidths up to 3 MHz can be displayed through the high speed D/A converters, and yet single block transforms can be sent to the computer for storage or additional processing.

Although the FFT was neither designed for nor optimized for

the application, a system was configured to demonstrate the FFT as a special purpose processor for the computer. Data from the computer is transformed by the FFT and the transform sent back to the computer. The computer input-output rate generally limits the speed of this operation, but considerable time savings are gained over the calculation time in a general purpose machine. If input-output word rates of 3 MHz were available, processing rates of less than 100 μ sec per 512 point transform are possible.

As an example of the spectrum analysis operation, examine the photographs in Figures 13 and 14. These were taken in a non-real time processing mode with the computer supplying the data to be transformed.

Adaptive Filtering

The 12 stage digital filter has strong potential in the area of adaptive filtering. Two features make it attractive: the filter characteristic is easily changed because of the high speed digital storage, and its organization permits the filter to operate completely asynchronously with the input signal. Its capability also includes continuous operation on channel IF bandwidths approaching 3 MHz and implementation of filters with up to 32 points of impulse response.

The following five part experiment was implemented to show how the system could be used as an adaptive processor. It is desired to correct for the distortion resulting from the transmission of a signal through an unknown channel. The channel frequency response

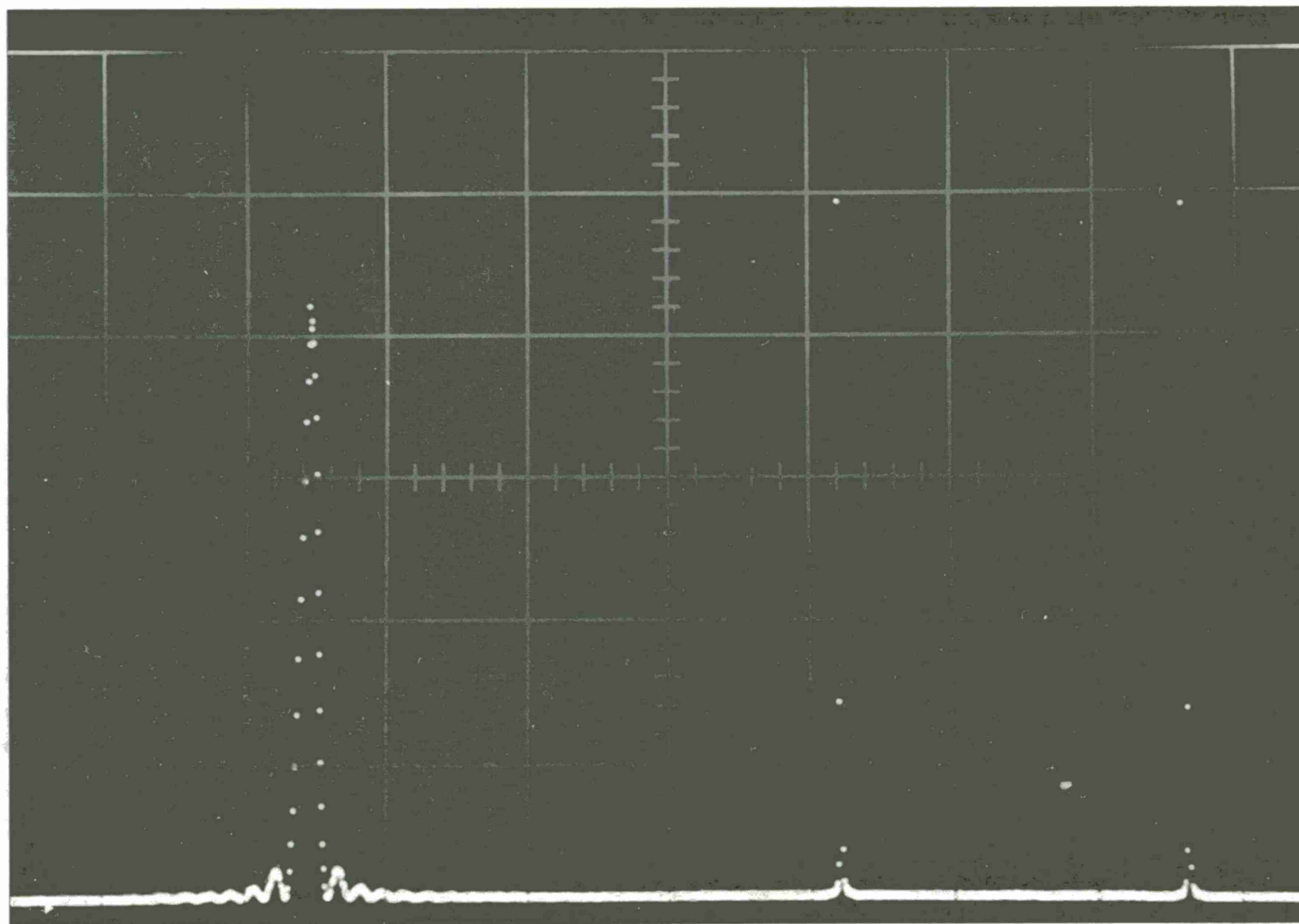


Figure 13. Squared Magnitude Output of 512 Point FFT for Video Pulse at Input #1 (Left Half of Display) and a Sinusoid at Input #2 (Right Half of Display)

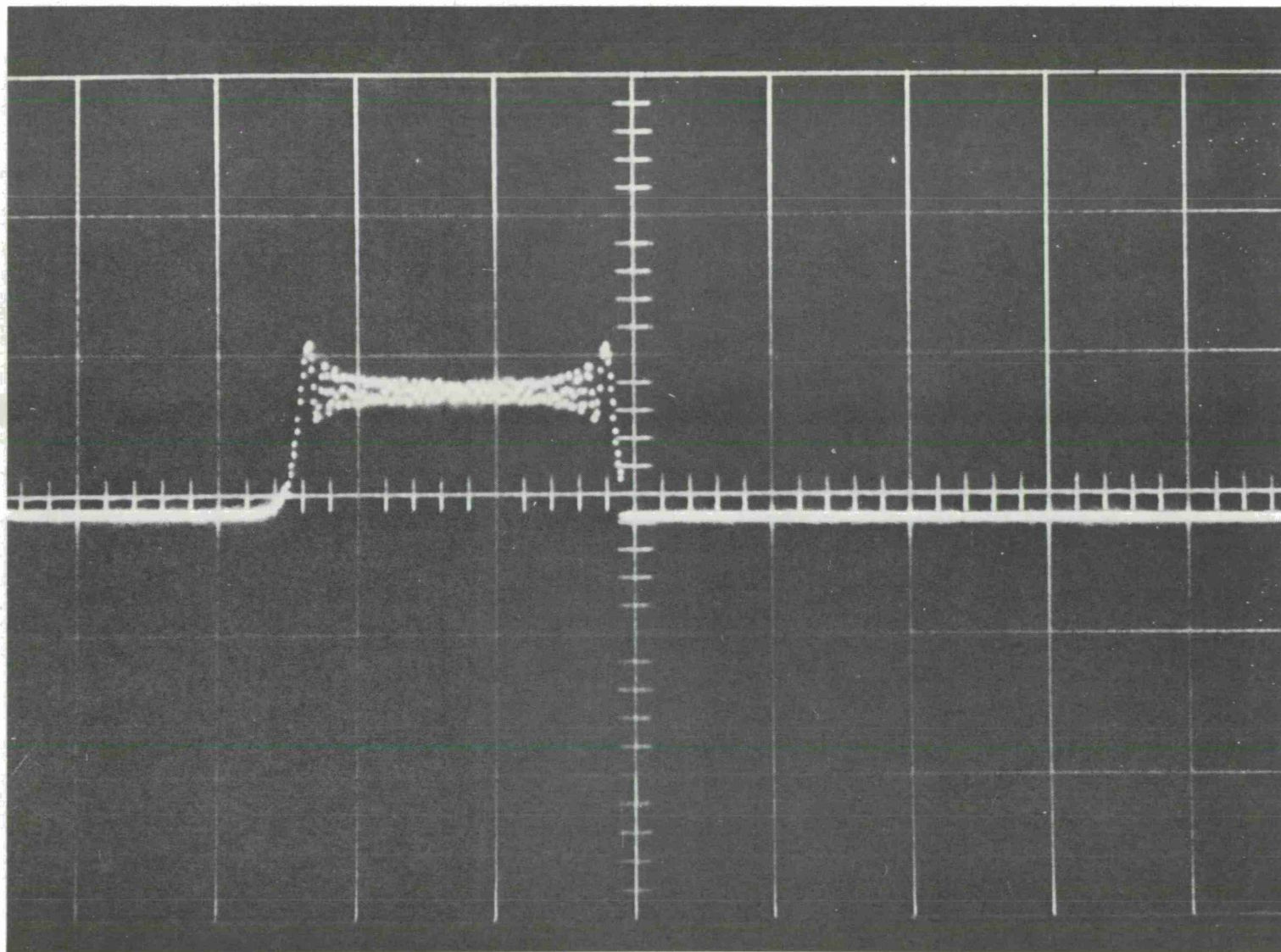


Figure 14. Squared Magnitude Output of 512 Point FFT for a Linear FM Signal at Input #1 (Left Output) and No Signal at Input #2 (Right)

is obtained by transforming the channel impulse response. The reciprocal of the frequency characteristic is then calculated. This reciprocal characteristic would be an ideal correction filter if it corresponded to a finite impulse response filter. Since in general it does not, it must be transformed into the time domain, truncated, and then transformed back into the frequency domain to give the desired characteristic. The detailed steps of the experiment are as follows. Also, see Figure 15.

- 1: Input to the digital filter 64 points of impulse response $e_s(k)$ from a distorted channel. Send the 64 point DFT, $E_s(n)$, (through the buffer memory if necessary) to the computer, where the reciprocal function $H_s(n) = \frac{1}{E_s(n)}$ is calculated.
- 2: Enter the 64 points of $H_s(n)$ into the filter characteristic memory.
- 3: Impulse the digital filter to generate the 64 point response $h_s(k)$.
- 4: Enter $h_s(k)$ into the filter input which blocks $h_s(k)$ into two separate 64 point functions, the first 32 and the last 32 points of $h_s(k)$ each padded out with 32 zero points. Store in the filter memory the 64 point transform $H_T(n)$ of the first 32 points of $h_s(k)$ padded out with 32 zeros. The desired filter characteristic is now stored.

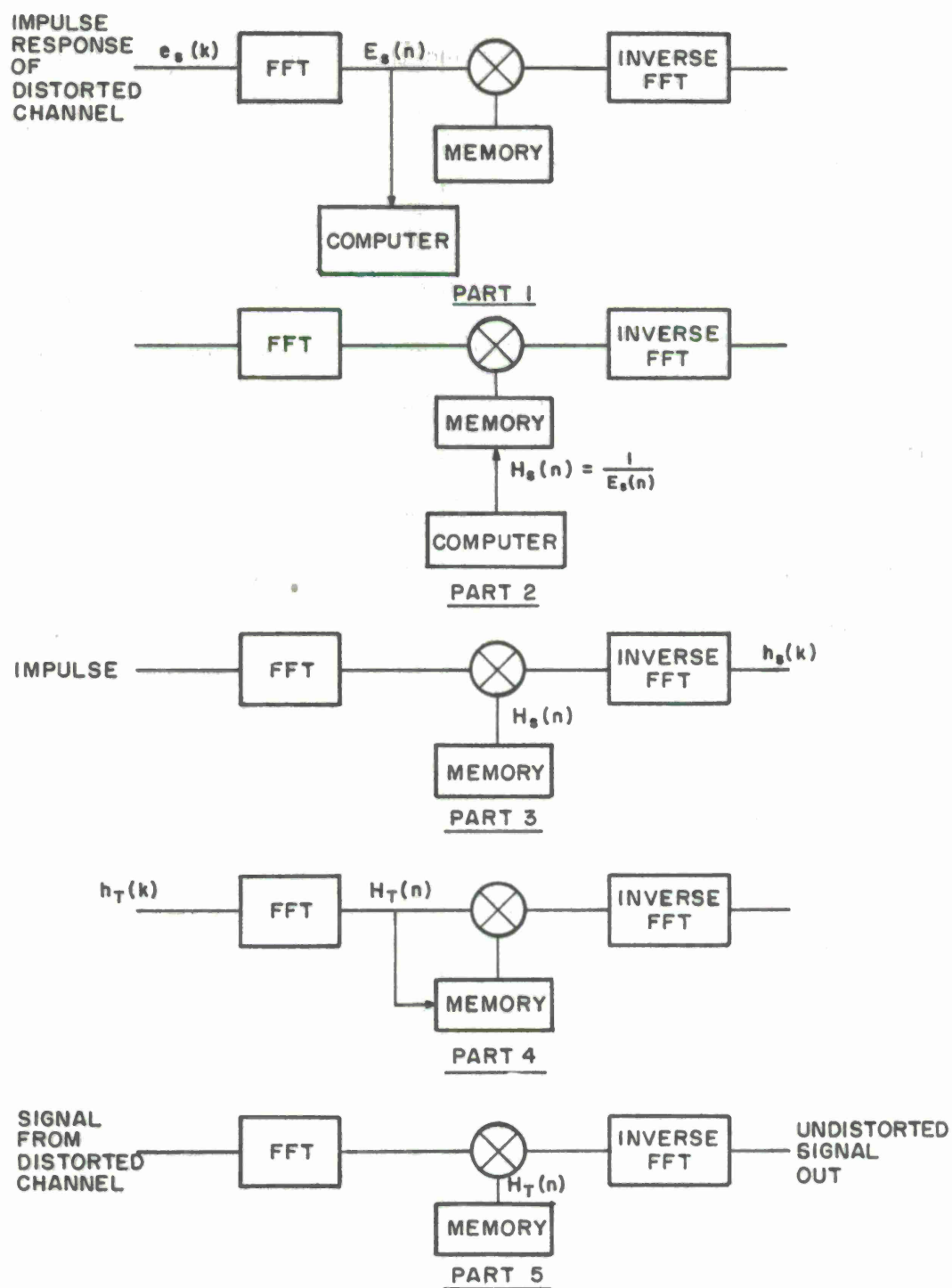


Figure 15. Adaptive Filter Experiment

5. Input $e_s(k)$ into the filter and the output should approximate an impulse function.

Parts 1 through 4 are the operations required to adapt the filter to a distorted channel, and the elapsed time for these parts is dependent on the computer I/O rates and its calculation time for $H_s(n)$. The processing rates of the filter is retained at the 3 MHz capability, since the computer is necessary only in adapting process.

For the demonstration experiment the computer served as both the controller as described earlier and as the signal generator. See Figure 16 for an example of the adaptive processor in operation.

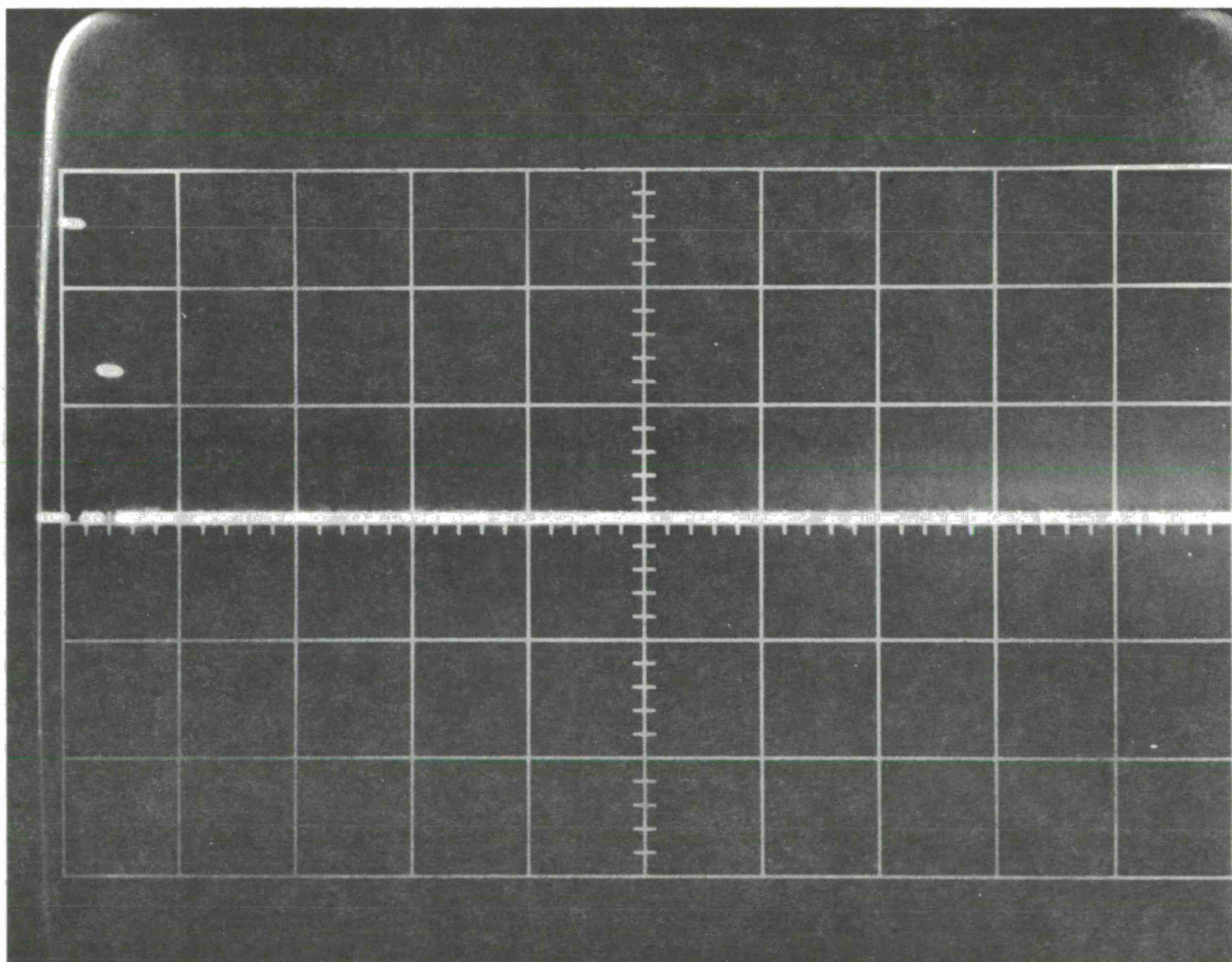


Figure 16a. Impulse Response of Distorted Channel With One Multipath

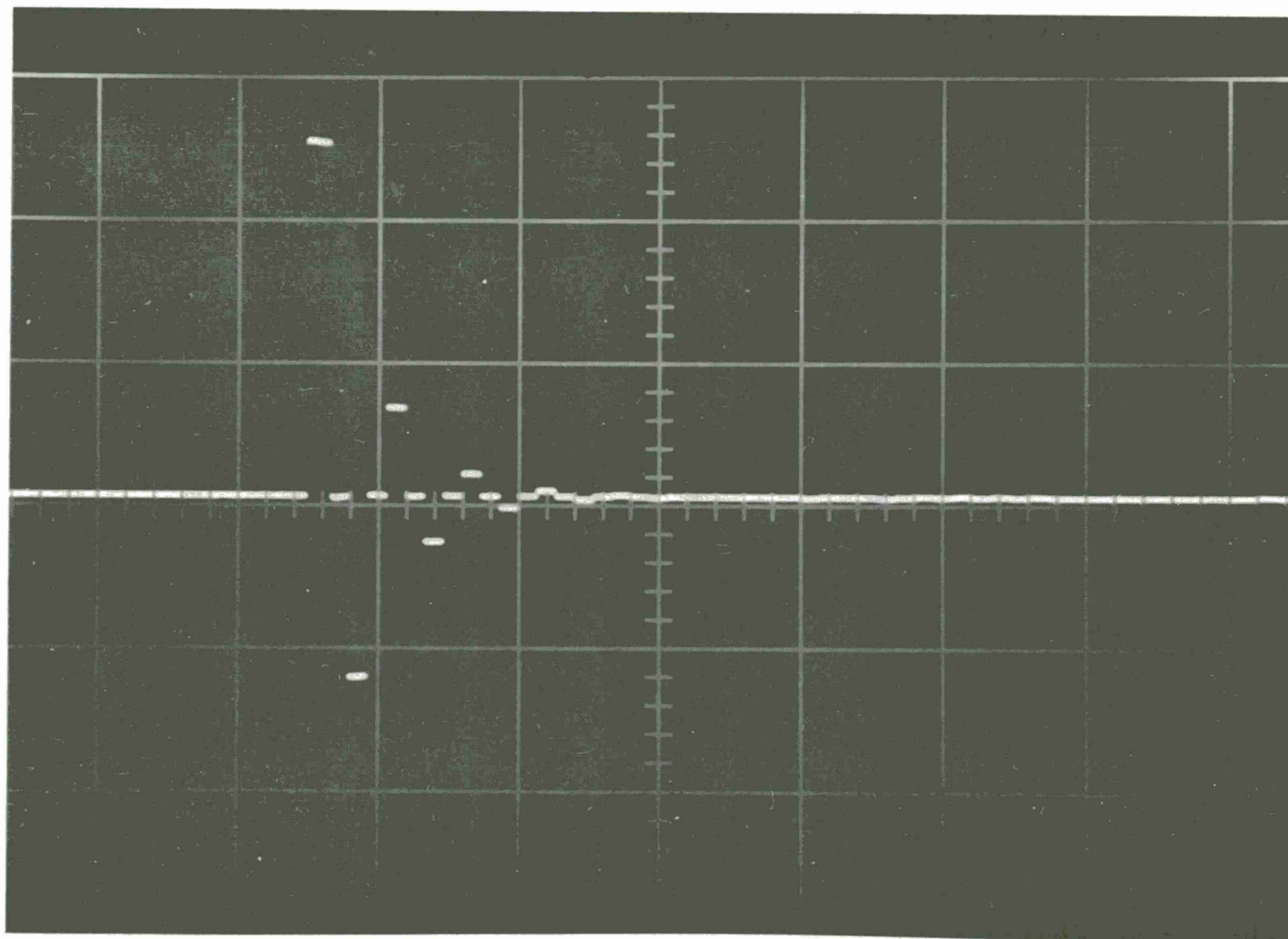


Figure 16b. Impulse Response of Correcting Filter From Part 3
of Adaptive Filter Experiment

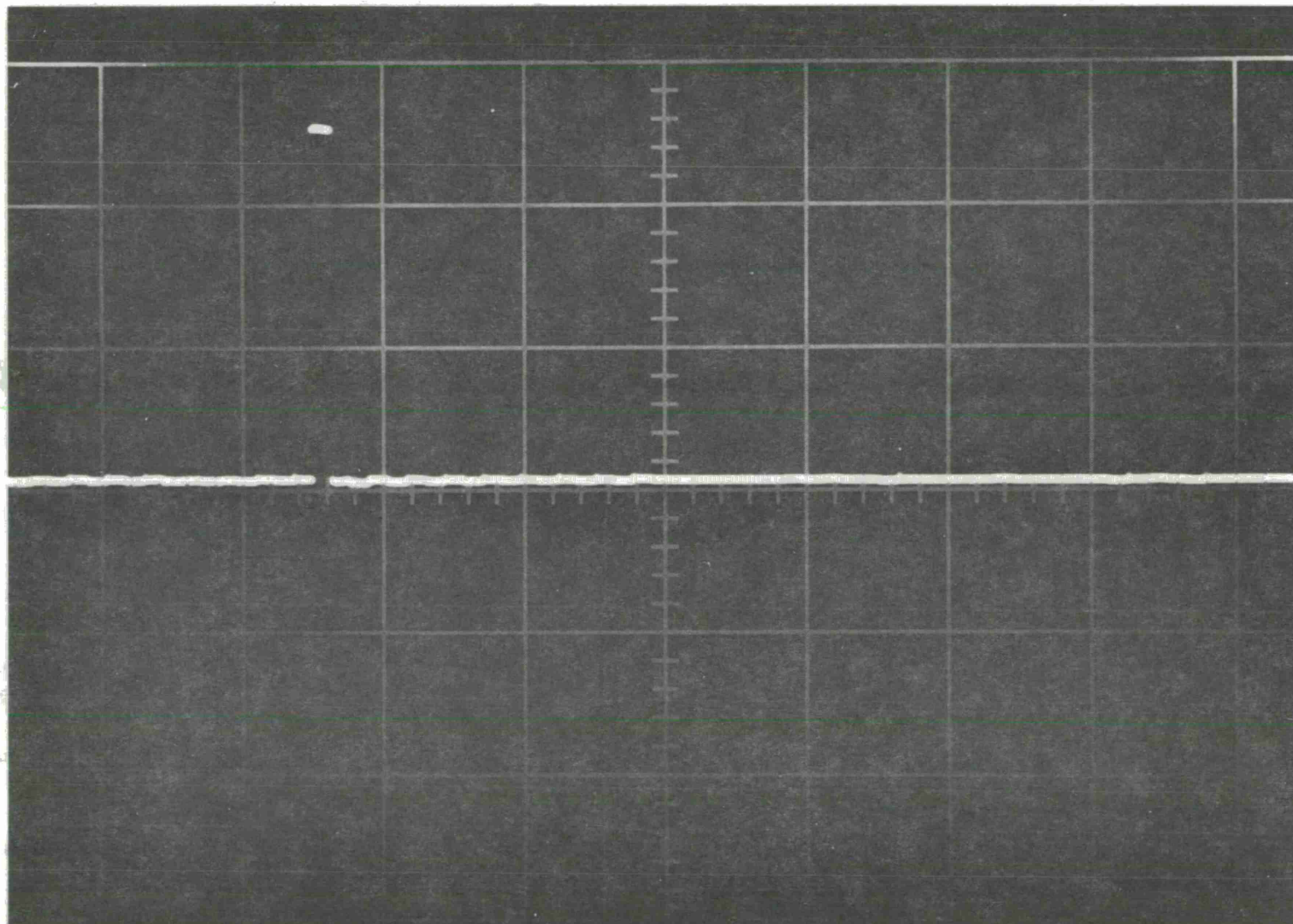


Figure 16c. Corrected Response From Part 5 of Adaptive Filter Experiment

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13. ABSTRACT <p>A hardwired digital processor based on the Cooley-Tukey algorithm is presented. A laboratory prototype has been built with two modes of operation. As a cascade fast Fourier transformer, it can simultaneously calculate transforms of two independent, continuous data streams at word rates in excess of 3 MHz. As a nonrecursive digital filter, it can produce filter impulse responses of up to 32 points. The digital filter also operates from data source with word rates exceeding 3 MHz.</p> <p>The processor has been integrated into a system with other signal processing components including a small general purpose computer. Laboratory demonstration of the processing system as a spectrum analyzer and as an adaptive filter for distortion correction is discussed.</p>			

14.

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